

# An $SO(4,1)$ group theory of quantum gravity

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## Abstract

The search for a quantum theory of gravity has become one of the most well-known problems in theoretical physics. Problems quantizing general relativity because it is not renormalizable have led to a search for a new theory of gravity that, while still agreeing with measured observations, is renormalizable. In this paper, I show that, given a “vortex” model of elementary particles in which rest mass derives from intrinsic spin and polarization, a Yang-Mills force with a  $SO(4,1)$  group symmetry predicts post-Newtonian N-body motion such as solar system observations of gravitational behavior as well as binary pulsar precession and orbital speed-up caused by gravitational radiation-reaction. I give a definition of the Yang-Mills theory on a lattice graph such that a background manifold is not required to define the theory. Using a homogeneous, isotropic universe model, I show that this theory does not contradict cosmological observations of Type 1a supernovae, Baryon Acoustic Oscillation, and the Cosmic Microwave Background. Most importantly, it agrees with the accelerating expansion of the universe as a consequence of the de Sitter group Lie algebra—an acceleration that I show does not occur in the Poincaré group approximation—suggesting that the de Sitter group symmetry explains dark energy. In addition, because it is a generic massless, semi-simple Yang-Mills theory, it is mathematically proved to be a perturbatively renormalizable quantum theory.

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## I. INTRODUCTION

That general relativity is able to subsume Newtonian gravity and explain phenomena that do not fit into the Newtonian framework such as Mercury's perihelion precession, light deflection, and gravitational red-shift as well as being compatible with special relativity has brought it wide acceptance. In recent years, however, as measurement tools have grown more accurate and new observations made, the necessity of introducing either tunable parameters or exotic forms of matter and energy to make general relativity fit those measurements has brought the theory into question. Observations of galactic rotational curves and gravitational lensing [1] [2] [3] have demanded the introduction of dark matter and accelerating expansion of the universe [4] dark energy. At present there is no consensus on what these substances are, whether they even exist, or whether they are features of a more complete theory of gravity.

Besides the problems with general relativity at the macroscopic scale, achieving a quantum theory of gravity has become one of the most significant unsolved problems in physics. Attempts to place the Einstein-Hilbert action,

$$S_{EH} = \int d^4x \sqrt{-g} R, \quad (1)$$

where  $R$  is the Ricci scalar, into a functional path integral ensemble, have all failed, giving nonsense results. The immediate source of the difficulty is that the theory is not renormalizable creating an infinite number of counterterms in perturbation expansions. Another, less theoretically troubling but mathematically difficult issue is that the expansion of the Lagrangian fails to terminate because of the volume element  $\sqrt{-g}$  and the inverse metric  $g^{\mu\nu}$ . Therefore, unlike the actions of other forces, the gravitational action is not finite polynomial. A similar disaster occurs quantizing the weak force but disappears when it is unified with the electromagnetic force, motivating the quest to unify gravity with the other forces in the hope that it will become renormalizable [5]. This unified theory, however, would not solve all of gravity's problems because, by design, it would become general relativity at galactic and cosmological scales where dark matter and energy have an effect. Therefore, a unified theory would not necessarily solve these large scale, classical problems without additional, low-energy features.

Although elegant and surprisingly accurate until recent decades, because the curved spacetime assumption causes such deep, unresolvable problems at the quantum level, there

is some motivation to find an alternative to spacetime curvature as the source of gravity. Despite its current acceptance, curved spacetime was a counterintuitive choice for a theory of gravity given the success of electromagnetism in the 19th century. As Wald points out [6],

Maxwell's theory is a remarkably successful theory of electricity, magnetism, and light which is beautifully incorporated into the framework of special relativity. Therefore, one might expect that the next logical step would have been to develop a new theory of the other classical force, gravitation, which would generalize Newton's theory and make it compatible with special relativity in the same way that Maxwell's theory generalized Coulomb's electrostatics. However, Einstein chose an entirely different path[.]

Because curved spacetime has never been precisely measured, the path not taken still lies open.

Indeed, we can ask how precisely general relativity has been confirmed, and, as this paper explores in detail, the answer is two-fold: (1) if general relativity's field equations were as simple as Maxwell's equations or not very much more complicated, then it could be considered a well-confirmed theory because its linearized version has a great deal of evidence behind it, and (2) although observation has confirmed many effects, many of the highly non-linear portions of the theory have been unobservable in any precise way because of the weakness of gravity. All precision experiments (classic tests) of general relativity have been done within the solar system (with the exception of binary pulsar precession) where spherically symmetric, weak gravity prevails and only the first order post-Newtonian Einstein-Infeld-Hoffman (EIH) equations have been confirmed [7] [6]. An example of the best recent evidence for strong field general relativity is the periastron precession of the double pulsar system PSR J0737-3039A/B [8], which is a first post-Newtonian order confirmation. Furthermore, recent measurements of the cosmic microwave background, where spacetime curvature is most likely to appear, show none at all [9] with the inflationary theory as a potential explanation [10].

The difference between the EIH equations and the full equations of GR has invited a large number of alternative theories of gravity such as are discussed in [11], but virtually all are metric theories and retain the curved spacetime picture with the associated problems

mentioned above. Non-metric theories, by contrast, offer the hope of eliminating gravity's problems with quantization at the cost of also eliminating the elegant curved spacetime approach. It has been suggested that the Standard Model approach, where forces have Yang-Mills actions, may be more suitable for a quantum theory of gravity because of its success at explaining the other forces. The tetrad or vierbein formulation of the Einstein-Hilbert action, for example, has a structure similar to the Yang-Mills [5]. Conformal gravity has a Yang-Mills structure as well but tends to diverge strongly from Newtonian gravity over large length scales [12]. None of these theories have achieved significant success in making predictions that general relativity is not capable of making nor in resolving, entirely, the problems with the quantum theory. Thus, there has been, as yet, no motivation for replacing general relativity with any of these nor of accepting any as quantum gravity's true representation.

In this paper, I present a Yang-Mills theory of gravity with a  $SO(4,1)$  or de Sitter group without a background metric. Because a non-Abelian  $SO(4,1)$  theory has local Lorentz symmetry, the theory complies with the Einstein Equivalence Principle (EEP) and the Strong Equivalence Principle (SEP) (which even most alternate metric theories do not satisfy). All physical theories are Poincaré symmetric, but, given that the Poincaré group is not semi-simple, the de Sitter group is a natural choice of a semi-simple extension. This paper represents the first time that a Yang-Mills theory based on this group has been shown to agree with precise gravitational observations.

Yang-Mills theories in general have a finite, polynomial action with a *renormalizable* quantization provided the coupling constant has non-negative mass dimension. Since  $G$  has a mass-dimension of  $-2$ , we turn to an unknown non-dimensional gravitational constant,  $a_g$ , for quantum predictions. Because the coupling constant is non-dimensional, mass, energy, momentum, and angular momentum are all quantized, which has already been suggested in the context of other theories of gravity [13]. Indeed, any quantum field can be decomposed into finitely many modes,

$$\phi = \sum_{n=-M/2}^{M/2} a_n e^{ik_n^\mu x_\mu}, \quad (2)$$

which constrains the number of degrees of freedom of the field to be a finite number,  $M$ . The quantity  $M$  is simply a regularization (energy cutoff) which is taken care of by renor-

malization, and, if the  $k_n$  are closely spaced then, we can take the continuum approximation,

$$\phi(x_\mu) \approx \int_{-M/2}^{M/2} \frac{dk^\mu}{(2\pi)^4} \hat{\phi}(k^\mu) e^{ik^\mu x_\mu}. \quad (3)$$

This is an approximation only however, similar to the thermodynamic limit approximation of classical fluids, taking the number of molecules to infinity, and, at a fundamental level, the number of degrees of freedom of the field remains finite.

With a lattice method, I show that the background coordinate system normally present in Yang-Mills theory can be replaced with an artificial, “bookkeeping” coordinate system where all distances, time intervals, and torsions in spacetime are derived from the Yang-Mills potential. In this way, the Yang-Mills action is the same as the standard Yang-Mills action for SU(N) theories, but its potential alone is the metric of spacetime.

Rest mass is derived from the energy of spin and polarization and this is the second component to the theory. In any frame, these vectors form an anti-symmetric tensor  $S^{\mu\nu}$  such that polarization is  $p^i = S^{0i}$  and spin is  $s^i = \epsilon^{ijk} S^{jk}$  where summation is implied. Spin and polarization have a conserved current (similar to the symmetric electromagnetic stress energy tensor [14]),

$$T^{\mu\nu} = - \left[ S^{\mu\lambda} S_\lambda{}^\nu + \frac{1}{4} \eta^{\mu\nu} S_{\alpha\beta} S^{\alpha\beta} \right], \quad (4)$$

where  $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$  is the Minkowski metric. This tensor is both symmetric and traceless. This implies that mass derives relativistically from an intrinsic kinetic energy of spin and oscillation.

With these definitions, this Yang-Mills theory, called *vortex gravity* because of the vortex particle interpretation that spin and oscillation implies, predicts N-body observations such as pertain to the Solar System and binary pulsars including radiation reaction, all with a finite polynomial, positive definite action that is renormalizable.

I show that the theory offers a cosmology slightly different from the  $\Lambda$ CDM dual epoch model. In this cosmology, I make a distinction between Doppler and gravitational redshift, i.e. redshift caused by matter moving within space and redshift caused by the expansion of space itself. For the expansion of matter, I derive a linear coasting cosmology. The linear cosmology has been shown to fit observations such as Type-Ia supernovae and the Cosmic Microwave Background related to the later universe well [15] [16]. In addition, I derive a gravitational potential which determines the degree of gravitational redshift. This leads to an interesting result: while in the Poincaré group approximation of the de Sitter

theory, the linear coasting model dominates the later universe with the gravitational redshift vanishing, in the de Sitter theory the combination of the Doppler and gravitational redshift predicts a linear acceleration in total observed redshift. I explain this phenomenon by observing that, in the de Sitter Lie algebra, momentum produces spin by the coupling of momentum generators  $[V^\mu, V^\nu] = iM^{\mu\nu}$  where  $V^\nu$  are momentum generators and  $M^{\mu\nu}$  are spin generators. In the Poincaré group spin can only be produced if spin is already present because  $[V^\mu, V^\nu] = 0$ . The ability of the de Sitter theory to produce spin from momentum without a source of spin causes the expansion of the universe to produce gravitons that couple to spin which in turn create gravitons that couple to momentum. The back and forth production of gravitons leads to an accelerating redshift. This explanation is similar to the explanation for Thomas precession where the orbital motion of an electron increases its spin—an anomaly explained in the early 20th century which can be shown from the Lorentz Lie algebra [14]. Hence, “dark energy” can be construed to be gravitons produced because of the relationship between momentum and spin in a de Sitter symmetry group. No cosmological constant is required, and no vacuum energy is modelled. The effect is produced directly from the Yang-Mills equations. Observations currently attributed to dark matter such as galactic rotational curves and anomalous lensing are not addressed in this paper.

The paper is organized as follows: Sec. II A describes the conserved quantities of mass and linear momentum, oscillation momentum, and angular momentum, how these are represented in the theory, and how they appear intrinsically through the vortex model; Sec. II B derives the Yang-Mills theory with only a lattice and the gravitational potential and no background manifold. This section also explains how the bookkeeping coordinate system works; Sec. II C shows a derivation of the field equations for the  $SO(4,1)$  Yang-Mills theory and its formulation as two covariant field equations; Sec. II E derives the equation for particle motion and shows that it is the same as the geodesic equation; Sec. II F gives a discussion for how the theory satisfies the Strong Equivalence Principle (SEP) which GR also satisfies; Sec. V discusses novel predictions made by the theory; Sec. II G briefly mentions the mathematical justification for a renormalizable quantization; Sec. III shows that the theory agrees with measurements within the solar system including redshift, perihelion precession, time dilation and light bending (III B), binary pulsar precession (III C), the quadrupole formula for gravitational radiation as well as showing that there are no dipole/monopole moments (III C 1), and the equations for an expanding Robertson-Walker universe (III D); Sec. IV

explores any possible experimental disagreement with the theory; Sec. V discusses tests that may confirm or deny the theory's predictions; and Sec. VI discusses related work. There are also two Appendices.

## II. THE THEORY

Although ad hoc gravitational theories exist, any strong theory tends to be based on a guiding principle. In developing this theory, the guiding principle is that gravity is an ordinary force in the standard,  $SU(3) \times SU(2) \times U(1)$  model. This does not imply, as is frequently mentioned in the literature, that gravity needs a background manifold to be defined as a non-metric theory. I show that it does not provided one accept a lattice graph construction to spacetime. In some sense, the lattice graph is no more presumptive in a discrete sense than Einstein's differential manifold and point-coincidence assumptions are in a continuous sense [17]. They both simply assume that events take place at points and that points have neighboring points. Indeed, the lattice version is the weaker of the two because it does not require continuity meaning that neighboring points need not be measurably close. An essential feature of the following theory is that all the physical requirements are supported in the literature, and all have physical explanations based on well-established principles derived from experiment.

### A. Conserved quantities of the vortex model in the classical limit

The vortex model of elementary particles was first introduced by Lord Kelvin for atoms and since expanded with modern intrinsic quantum properties of particles [18]. Even pre-dating the development of special relativity, the mass of the electron has been presumed to derive from its electrical energy [19], and hence the stress-energy tensor of an electron, and, by extension to quarks and other particles, all matter, must, classically, take the general form of the Maxwell stress-energy tensor. For neutral particles, their stress energy is based on spin and oscillation rather than magnetic and electric fields, but the critical point is that spin/oscillation has, not only mass, but kinetic energy in the form of static pressure. In other words, particles cannot be represented as "point masses", blobs of matter where the only non-zero component of their stress-energy tensors is  $T^{00} = m$  (in the rest frame). The

stress-energy tensors have space-space components as well,  $\delta_{jk}T^{jk} = m$ . Because they are positive, these space-space components cannot be cancelled out in scaling up to macroscopic bodies.

The vortex model proposes that all mass derives from intrinsic quantum properties of spin and oscillation. Spin and oscillation are not separate entities, and each particle has a covariant, anti-symmetric spin tensor,  $s^{\mu\nu} = -s^{\nu\mu}$ , forming a single 4-dimensional spin in the six planes of a Minkowski space. In any frame, oscillation occurs in the direction of the polarization vector  $P^i = s^{0i}$  and spin in the plane with spin vector  $S^i = \epsilon^{ijk}s^{jk}$ .

The  $5 \times 5$  matrix current of the SO(4,1) Yang-Mills theory,  $J_\mu$ , describes the motion of particles (see A for an overview of the de Sitter group). This current, written in component form, is,

$$J_\mu = T_{\mu\nu}(V^\nu) + S_{\mu\alpha\beta}M^{\alpha\beta}, \quad (5)$$

where  $S_{\mu\alpha\beta}$  represents Lorentz boosts (oscillations) and rotations (total angular momentum), and  $T_{\mu\nu}$  represents the stress energy momentum tensor which is conserved by translations, where  $\mu, \nu = 0, 1, 2, 3$ .

I define the conserved currents in terms of spin and polarization[35] vectors. Classical particles oscillate and spin at the speed of light,  $c = 1$ ; therefore, we have rest frame polarization vector  $p^i$  and rest frame spin vector  $s^i$ .

From the spin/polarization vectors,  $s^i$  and  $p^i$ , a tensor is formed which can then be in any frame,

$$S_{\mu\nu} = \begin{pmatrix} 0 & p^1 & p^2 & p^3 \\ -p^1 & 0 & s^3 & s^2 \\ -p^2 & -s^3 & 0 & s^1 \\ -p^3 & -s^2 & -s^1 & 0 \end{pmatrix}. \quad (6)$$

The first current, total energy, is given in any frame by,

$$T_\mu{}^\nu = -[S_{\mu\lambda}S^{\lambda\nu} + \frac{1}{4}\delta_\mu{}^\nu S_{\alpha\beta}S^{\alpha\beta}]. \quad (7)$$

Like the electromagnetic stress energy tensor, this has the form (in Cartesian coordinates



$(x, y, z))$  [14],

$$T_{\mu\nu} = \begin{pmatrix} \frac{1}{2}(p^2 + s^2) & U_x & U_y & U_z \\ U_x & -P_{xx} & -P_{xy} & -P_{xz} \\ U_y & -P_{yx} & -P_{yy} & -P_{yz} \\ U_z & -P_{zx} & -P_{zy} & -P_{zz} \end{pmatrix}, \quad (8)$$

where  $\vec{U} = \vec{p} \times \vec{s}$ ,  $P_{ij} = p_i p_j + s_i s_j - \frac{1}{2}(p^2 + s^2)\delta_{ij}$ ,  $p^2 = (p_x)^2 + (p_y)^2 + (p_z)^2$ , and  $s^2 = (s_x)^2 + (s_y)^2 + (s_z)^2$ .

Similar to the electromagnetic Poynting vector,  $\vec{S}_{\text{Poynting}} = \frac{1}{2}\vec{E} \times \vec{B}$ , the vector  $\vec{U}/m$  is a particle's flux. The energy flux is  $\vec{U}$ . Particles in the rest frame have no energy flux. This gives a definition of a massive particle's rest frame: it is the frame where the polarization and spin vectors are parallel,  $\vec{p} \times \vec{s} = 0$ . This means that in every particle's rest frame, the plane of oscillation is orthogonal to the plane of spin. Classically, one can visualize this as a point particle pursuing a periodic helical motion revolving about its axis of spin while oscillating normal to the plane of spin like a tiny vortex. As the particle increases velocity, the angle between the planes decreases until they become parallel at the speed of light. Therefore, the two 3-vectors are plus or minus the same direction,  $p^i = \pm s^i$ .

The conserved current for massive particles in the rest frame has a convenient representation in spherical coordinates,

$$T_{00} = T_{rr} = (p^2 + s^2) \quad (9)$$

Because  $p^2 + s^2 = m$  (with  $c = 1$ ) in the rest frame,

$$T_{00} = T_{rr} = m \quad (10)$$

Matter and antimatter both have positive relativistic mass (as do photons) as a natural consequence of the positiveness of the polarization and spin energy. This energy does not change its sign. For example, in a conversion experiment from a electron-positron pair in a head-on collision to a pair of photons emerging in opposite directions,  $e^- + e^+ = 2\gamma$ , because the total momenta sum to zero (neglecting spins), the relativistic masses going in must sum to those coming out. Hence,

$$T_{00}^{e^-} + T_{00}^{e^+} = 2T_{00}^{\gamma}. \quad (11)$$

Because electrons and positrons have the same relativistic mass if travelling at the same speed, we have  $T_{00}^{e^-} = T_{00}^{\gamma}$ . This implies that matter, antimatter, and photons are all

gravitationally attracted to one another and, further, that antimatter behaves the same way in a gravitational field as matter. (This result is already predicted by general relativity.)

Finally, the conserved currents described here should not be confused with the definition of stress-energy-momentum in general relativity. General relativity provides a definition of energy based on the symmetries of spacetime geometry. If we have a Lagrangian that depends on the metric field,  $\mathcal{L}(g_{\mu\nu})$ , then energy is simply the variation of that Lagrangian with respect to the field,

$$\frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} = T^{\mu\nu}. \quad (12)$$

This is not the tensor,  $T^{\mu\nu}$ , we have been discussing above. General relativity assumes diffeomorphism-covariance which implies that the quantity 12 is conserved by Noether's theorem. Kretschmann pointed out to Einstein himself (and Einstein conceded), however, that general covariance, i.e. coordinate covariance, was a fictitious symmetry group generating no physical predictions [17]. Many theories can be made that are generally covariant and predict different physical outcomes. The physical restrictions on the theory are important and fix the theory. Diffeomorphism-covariance in general relativity is an active symmetry that is unrestricted. This is not so in the Yang-Mills theory because the gauge prefers a particular coordinate system. In other words, once the gauge is fixed it specifies a specific set of reference frames at each spacetime location. With a preferred set of reference frames, although the coordinate system can be changed at will, the theory is not actively diffeomorphism-covariant. Hence, Noether's theorem does not apply to the metric, and the stress-energy-momentum tensor of General Relativity is not a conserved quantity. This is not the same as having a preferred coordinate system overall, only that fixing the gauge of the Yang-Mills theory destroys the diffeomorphic gauge freedom that general relativity assumes. In the next section I will show this mathematically.

## B. Derivation of a Yang-Mills theory without a background metric

Yang-Mills theory is typically defined with a Minkowski background metric,  $\eta_{\mu\nu}$ . In this section, I derive a Yang-Mills theory without a background manifold using a dual labelling approach where I associate each vertex on a four dimensional lattice with two labels: (1) 4-vector,  $y_a = (y_0, y_1, y_2, y_3)$ , labels derived from the gravitational potential that represent real distances and torsions between vertices and (2) a second set of labels,  $x_a = (x_0, x_1, x_2, x_3)$ ,

that are equidistant between neighboring vertices. The mapping  $x_a \rightarrow y_a$  is one-to-one and onto since there is exactly one of each label for each vertex. At no time does a background need to be invoked, but, when I take the continuum limit, a metric appears. In this section only, I will use the beginning letters of the latin alphabet  $a, b, c, \dots, h$  to represent 4-vectors. (I prefer not to use the usual Greek letters  $\mu, \nu$  because they are typically used with a metric and a diffeomorphic manifold. Here we assume no general covariant structure because there is no manifold.) These letters range over  $0, 1, 2, 3$ . I will use  $y_4$  and  $x_4$  to refer to the additional, 5th dimension that completes de Sitter group vectors. I will use the middle latin letters  $i, j, k, l$  (in this section only) to refer to entire vectors. When I do so, the vectors will be bolded, e.g.,  $\mathbf{y}_i$ .

Consider a four dimensional lattice and label each vertex with four numbers  $y_0, y_1, y_2, y_3$ . We do not yet know what these labels are. Let time be imaginary  $t \rightarrow i\tau$ . We will analytically continue our result back to  $SO(4, 1)$  later. Consider three vertices in a row  $y, z$ , and  $w$ , let  $U_{yw}(z)$  be a *small* rotation matrix with  $\epsilon > 0$  the small parameter.  $U_{yw} = \exp(\epsilon A_a)$  can be split into two parts:  $G_{ab}$  and  $H_{abc}$  by the following: an  $SO(5)$  potential can be written as a sum of generators:

$$A_a = G_{ab}V_b + H_{abc}M_{bc}, \quad (13)$$

where  $V_b$  are generators for rotations in the  $y_b$ - $y_4$  plane and  $M_{bc}$  are generators for  $SO(4)$  for rotations in the  $y_b$ - $y_c$  plane where  $a, b, c = 0, 1, 2, 3$ . We will refer to  $G_{ab}$  and  $H_{abc}$  as the *gravitational potentials*.

To linear order in  $\epsilon$ ,

$$U_{yw} = I + \epsilon A_a = I + \epsilon(G_{ab}V_b + H_{abc}M_{bc}).$$

Each of the generator matrices is a 5 by 5 matrix. Let  $\mathbf{y} = (y_0, y_1, y_2, y_3, y_4)$  and  $\mathbf{w} = (w_0, w_1, w_2, w_3, w_4)$ , then we define

$$\mathbf{w} := [I + \epsilon(G_{ab}V_b + H_{abc}M_{bc})]\mathbf{y}. \quad (14)$$

We need to define only one point in the entire lattice,  $\bar{y} = (0, 0, 0, 0, \bar{y}_4)$ , in order to define all the other points using this procedure.

Getting rid of the generator matrices, the equation 14 implies the relationship between the 4-vectors,

$$w_a = \epsilon H_{abc}y_c + \epsilon G_{ab} + y_b, \quad (15)$$

again, all to linear order in  $\epsilon$ . The first term in the right hand side of 15 represents the torsion or twisting of a vector as it travels from  $\mathbf{y}$  through  $\mathbf{z}$  to end at  $\mathbf{w}$ . The second term on the right hand side represents the translation. (Note: if  $G_{ab}(\mathbf{z}) = 0$  then there is no translation and the two lattice points are effectively in the same location. This means that it is possible for spacetime to collapse in on itself as in the Big Bang.)

Thus, the labels of the lattice,  $\mathbf{y}$ , which previously had no meaning, are now defined in terms of the gravitational potentials by hopping from point to point outward from the only pre-defined point  $\bar{y}$  (which is arbitrary of course). A similar procedure is used to define the natural numbers in terms of a successor function  $S$ , where 0 is the only predefined point, from Peano's axioms. The gravitational potentials,  $G$  and  $H$ , are successor functions that define the spacetime lattice.

Having defined the labels as coordinates in a discrete spacetime, we can derive Yang-Mills theory using Wilson's method for lattices. First, define the second labelling system by the same method but using trivial potentials  $G'_{ab} = \delta_{ab}$ , the Kronecker delta such that  $\delta_{aa} = 1$  and  $\delta_{ab} = 0$  for  $a \neq b$ , and  $H'_{abc} = 0$ . This labelling system is denoted by  $x_a$  and corresponds to a lattice where each vertex is a distance  $\epsilon/2$  from each of its six neighbors.

Consider a plaquette  $p$  between four vertices in the  $a - b$  plane between  $\mathbf{x}_i$ ,  $\mathbf{x}_j$ ,  $\mathbf{x}_k$ ,  $\mathbf{x}_l$ , and back to  $\mathbf{x}_i$ , and assume that each vertex is two lattice edges from its two neighbors. The action on the plaquette is given by  $S_p = \text{Re tr } U_{ij}U_{jk}U_{kl}U_{li}$ . Now,  $U_{ij} = \exp(\epsilon A_a(\mathbf{x}))$  where  $\mathbf{x} = (\mathbf{x}_i + \mathbf{x}_j)/2$  is the midpoint between  $\mathbf{x}_i$  and  $\mathbf{x}_j$  and likewise for  $U_{jk}$ ,  $U_{kl}$ , and  $U_{li}$  [5].

$$U_{ij}U_{jk} = e^{\epsilon A_a}e^{\epsilon A_b} = e^{\epsilon(A_a+A_b)+\frac{1}{2}\epsilon^2[A_a,A_b]+O(\epsilon^3)}$$

$$U_{kl}U_{li} = e^{-\epsilon A'_a}e^{-\epsilon A'_b} = e^{-\epsilon(A'_a+A'_b)+\frac{1}{2}\epsilon^2[A_a,A_b]+O(\epsilon^3)}$$

Now, some matrix algebra needs to be done and can be found in any standard text book on quantum field theory such as [5] or in Wilson's original paper [?] and we have,

$$U_{ij}U_{jk}U_{kl}U_{li} = e^{\epsilon^2\{(A_a-A'_a)/\epsilon-(A_b-A'_b)/\epsilon+[A_a,A_b]\}+O(\epsilon^4)}$$

which simplifies to

$$U_{ij}U_{jk}U_{kl}U_{li} = I + \epsilon^2 \left[ \frac{(A_a - A'_a)}{\epsilon} - \frac{(A_b - A'_b)}{\epsilon} + [A_a, A_b] \right] + \left( \epsilon^2 \left[ \frac{(A_a - A'_a)}{\epsilon} - \frac{(A_b - A'_b)}{\epsilon} + [A_a, A_b] \right] \right)^2,$$

to the required order. Summing over all plaquettes and we have a constant term from the first which we call the vacuum energy (note that here the gravitational potential does not

couple to vacuum energy), the middle term cancels out, and we are left with an equation for the total action,

$$S = \sum_p \text{Re tr} \left( \epsilon^2 \left[ \frac{(A_a - A'_a)}{\epsilon} - \frac{(A_b - A'_b)}{\epsilon} + [A_a, A_b] \right] \right)^2$$

Taking  $\epsilon \rightarrow 0$  gives us the Yang-Mills potential and continuing to real time from imaginary time changes the  $\delta_{ab}$  operators into  $\eta_{ab}$  operators,

$$S = \frac{1}{4g^2} \int d^4x \eta_{ac} \eta_{bd} F_{ab} F_{cd}, \quad (16)$$

where  $g$  is the coupling constant and

$$F_{ab} = \partial_a A_b - \partial_b A_a + [A_a, A_b]. \quad (17)$$

The Minkowski metric has appeared because we continued from an  $\text{SO}(5)$  group to a  $\text{SO}(4,1)$  group and our second labelling system is generated with  $G'_{ab} = \delta_{ab}$ . In real time, that generator becomes  $G'_{ab} = \eta_{ab}$ .

The so-called “background” metric of our theory, then, is the metric over an artificial labelling system  $x_a$ , while the true structure of spacetime is described by the first labelling system  $y_a$ . The artificial coordinate system is anything we choose it to be. It need not be Cartesian. The only requirement is that we have the ability to map every coordinate point  $x_a$  to every true coordinate point  $y_a$  in a one-to-one, onto fashion. Therefore, we have the freedom to choose coordinate systems that are useful.

It is important to take away from this section that the  $x_a$  coordinates, metric, and diffeomorphic symmetry on the coordinates are not real in the same way that the  $y_a$  coordinates, gravitational potentials, and locally de Sitter symmetry are real. The coordinate system,  $x_a$ , is generally covariant in the continuum limit, but the diffeomorphic symmetry is vacuous because the coordinate points *are not measurable*. Only the first coordinate system,  $y_a$ , is measurable. Distances and time can be measured with rods and clocks. Torsion can be measured by detecting particle spin and oscillation. Hence, the coordinates  $y_a$  are something an experimentalist can discover, but  $x_a$  can only be invented. Therefore, the  $\text{SO}(4,1)$  action 16 describes reality while the diffeomorphic coordinate system,  $x_a$ , describes a bookkeeping system imposed on reality.

### C. Yang-Mills theory and derivation of the field equations

In this section, I begin with standard Yang-Mills theory and derive a locally de Sitter theory of quantum gravity using the de Sitter Lie algebra. Yang-Mills theory describes forces as exchanges of gauge bosons based on a group symmetry. The Standard Model of quantum field theory currently contains three forces, electromagnetism, the weak force, and the strong force in a  $U(1) \otimes SU(2) \otimes SU(3)$  group symmetry.

A conventional  $SU(N)$  Yang-Mills theory has action,

$$S = -\frac{1}{4g^2} \int d^4x F_{\mu\nu} F^{\mu\nu}, \quad (18)$$

where  $g$  is the coupling constant and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \quad (19)$$

is the “force” where  $A_\mu$  is a matrix potential from the  $SU(N)$  group, written with indexes as  $A_{\mu ab}$ . For the  $SO(4,1)$  group these are  $5 \times 5$  matrices. (Note that in the previous section we absorbed a factor of  $i$  into  $A_\nu$  to make the matrices real. The two expressions for  $F_{\mu\nu}$  are equivalent.) The theory is invariant under a gauge transformation  $U(x)$  such that  $A'_\mu = U(x)A_\mu U^\dagger(x) - i/g \partial_\mu U(x) U^\dagger(x)$ . For infinitesimal transformations  $U(x) = e^{i\chi(x)} \approx I + i\chi(x)$  we have  $A'_\mu = A_\mu + \partial_\mu \chi(x) + i[\chi(x), A_\mu]$ .

Yang-Mills equations are often expressed in group component form, where the action is,

$$S = -\frac{1}{4g^2} \int d^4x F_{\mu\nu}^a F^{\mu\nu a}, \quad (20)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c, \quad (21)$$

and  $f^{abc}$  is the group symbol. Each component of the potential represents a field of spin-1 gauge bosons.

Rather than decomposing into pure component form which is not covariant, it is more useful to put it into a covariant form. The  $SO(4,1)$  potential can be written as a sum of generators of Lorentz rotations and boosts and of boost/rotations with respect to  $x_5$ ,

$$A_\mu = G_{\mu\nu} V^\nu + H_{\mu\nu\lambda} M^{\nu\lambda}, \quad (22)$$

where the potential  $G_{\mu\nu}$  couples to  $T^{\mu\nu}$  and  $H_{\mu\nu\lambda}$  to  $S^{\mu\nu\lambda}$ . In this sense, the theory resembles Einstein-Cartan theory.

Following the de Sitter algebra of A, the “forces” of gravity decompose into a covariant form with two equations analogous to the Einstein field equations and the Cartan torsion equations respectively,

$$E_{\mu\nu\lambda} = \partial_\mu G_{\nu\lambda} - \partial_\nu G_{\mu\lambda} + \eta^{\sigma\rho} (G_{\mu\lambda} H_{\nu\sigma\rho} - G_{\mu\rho} H_{\nu\sigma\lambda} - G_{\nu\lambda} H_{\mu\sigma\rho} + G_{\nu\rho} H_{\mu\sigma\lambda}), \quad (23)$$

and

$$F_{\mu\nu\alpha\beta} = \partial_\mu H_{\nu\alpha\beta} - \partial_\nu H_{\mu\alpha\beta} + G_{\mu\alpha} G_{\nu\beta} - G_{\mu\beta} G_{\nu\alpha} + \Phi_{\mu\nu\alpha\beta} - \Phi_{\mu\nu\beta\alpha} \quad (24)$$

where

$$\Phi_{\mu\nu\alpha\beta} = \eta^{\sigma\rho} (H_{\mu\sigma\alpha} H_{\nu\rho\beta} - H_{\mu\sigma\alpha} H_{\nu\beta\rho} - H_{\mu\alpha\sigma} H_{\nu\rho\beta} + H_{\mu\alpha\sigma} H_{\nu\beta\rho}). \quad (25)$$

Thus, the action of gravity is,

$$S_{gravity} = -\frac{1}{4a_g} \int d^4x E_{\mu\nu\lambda} E^{\mu\nu\lambda} + F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta}. \quad (26)$$

Here the coupling constant is  $a_g = g^2$ .

The classical equations of motion of any Yang-Mills theory can be found by the variation of the action,  $\delta S = 0$ , via the Euler-Lagrange equations,

$$\partial^\mu \left( \frac{\partial \mathcal{L}}{\partial (\partial^\mu A_\nu^a)} \right) - \frac{\partial \mathcal{L}}{\partial A_\nu^a} = 0, \quad (27)$$

where  $\mathcal{L} = F_{\mu\nu}^a F_a^{\mu\nu}$  is the Lagrangian. Evaluating the Euler-Lagrange equations for the Yang-Mills Lagrangian, the equations of motion are,

$$\partial^\mu F_{\mu\nu} - i[F_{\mu\nu}, A^\mu] = -8\pi g^2 J_\nu, \quad (28)$$

where  $J_\nu$  is the conserved current.

Applying the definition of the group symbol again and the previous definitions 23 and 24 to the field equations (28) yields the following equations:

$$\partial^\mu E_{\mu\nu\lambda} + \eta^{\sigma\rho} (E_{\mu\nu\lambda} H^\mu_{\sigma\rho} - E_{\mu\nu\rho} H^\mu_{\sigma\lambda} - G^\mu_{\lambda} F_{\mu\nu\sigma\rho} + G^\mu_{\rho} F_{\mu\nu\sigma\lambda}) = 8\pi T_{\nu\lambda} \quad (29)$$

$$\partial^\mu F_{\mu\nu\alpha\beta} + E_{\mu\nu\alpha} G^\mu_{\beta} - E_{\mu\nu\beta} G^\mu_{\alpha} + \Sigma_{\nu\alpha\beta} - \Sigma_{\nu\beta\alpha} = 8\pi S_{\nu\alpha\beta} \quad (30)$$

where

$$\begin{aligned}\Sigma_{\nu\alpha\beta} = & \eta^{\sigma\rho} (F_{\mu\nu\sigma\alpha} H^\mu{}_{\rho\beta} - F_{\mu\nu\sigma\beta} H^\mu{}_{\rho\alpha} - \\ & F_{\mu\nu\alpha\sigma} H^\mu{}_{\rho\beta} + F_{\mu\nu\alpha\beta} H^\mu{}_{\rho\sigma})\end{aligned}\quad (31)$$

and the usual negative sign on the coupling constant has been absorbed into the current in the same way that  $-i$  is absorbed into the potential. The coupling constant,  $a_g = g$ , is non-dimensional, but, because the size of the mass quantum is not known, I use the standard coupling constant,  $\kappa = G/c^4$ , in units such that  $\kappa = 1$ .

Boundary conditions are:

$$G_{\mu\nu} \rightarrow \eta_{\mu\nu}, \quad \partial_\lambda G_{\mu\nu} \rightarrow 0 \quad (32)$$

and

$$H_{\mu\nu\lambda} \rightarrow 0, \quad \partial_\alpha H_{\mu\nu\lambda} \rightarrow 0 \quad (33)$$

as  $x_\mu \rightarrow \infty$ .

Gauge transformations can be done in component form. Consider the  $SO(4, 1)$  de Sitter transformation  $\Delta(x) = \xi_\mu(x)V^\mu + \chi_{\mu\nu}M^{\mu\nu}$ . The gauge transformation is

$$G'_{\mu\nu} = G_{\mu\nu} + \partial_\mu \xi_\nu(x) + \eta^{\rho\lambda}(\xi_\lambda(x)H_{\mu\rho\nu} - \xi_\lambda(x)H_{\mu\nu\rho} + \chi_{\lambda\nu}(x)G_{\mu\rho} - \chi_{\nu\rho}(x)G_{\mu\lambda}) \quad (34)$$

and

$$H'_{\mu\nu\lambda} = H_{\mu\nu\lambda} + \partial_\mu \chi_{\nu\lambda}(x) + \xi_\nu(x)G_{\mu\lambda} + \eta^{\kappa\rho}(\chi_{\kappa\nu}(x)H_{\mu\rho\lambda} - \chi_{\kappa\nu}(x)H_{\mu\lambda\rho} - \chi_{\nu\kappa}(x)H_{\mu\rho\lambda} + \chi_{\nu\kappa}(x)H_{\mu\lambda\rho}). \quad (35)$$

#### D. Equations for evolution of matter

The continuity equations are standard to Yang-Mills theory. By Noether's theorem, for any matter action,  $\mathcal{S}_M$ , e.g., Dirac's, we have the relation between the potential  $G_{\mu\nu}$  and the stress-energy-tensor  $T^{\mu\nu}$  and the potential  $H_{\mu\nu\lambda}$  and the spin-density  $S^{\mu\nu\lambda}$ ,

$$T^{\mu\nu} = \frac{\delta \mathcal{S}_M}{\delta (G_{\mu\nu})}, \quad (36)$$

$$S^{\mu\nu\lambda} = \frac{\delta \mathcal{S}_M}{\delta (H_{\mu\nu\lambda})}, \quad (37)$$



and the continuity equations are,

$$D^\mu T_{\mu\nu} = 0, \quad (38)$$

$$D^\mu S_{\mu\nu\lambda} = 0 \quad (39)$$

[20], where for a generic source  $J_\mu^a$  and potential  $A_\mu^a$ ,  $D_\mu J_\nu = \partial_\mu J_\nu - i[A_\mu, J_\nu]$  is the covariant derivative. Hence the full continuity equations are,

$$\begin{aligned} \partial^\mu T_{\mu\lambda} + \eta^{\sigma\rho} (G^\mu{}_\lambda S_{\mu\sigma\rho} - G^\mu{}_\rho S_{\mu\sigma\lambda} - \\ T^\mu{}_\lambda H_{\mu\sigma\rho} + T^\mu{}_\rho H_{\mu\sigma\lambda}) = 0, \end{aligned} \quad (40)$$

$$\partial_\mu S^{\mu\alpha\beta} + G^\mu{}_\alpha T_{\mu\beta} - G^\mu{}_\beta T_{\mu\alpha} + \Pi^{\alpha\beta} - \Pi^{\beta\alpha} = 0 \quad (41)$$

where

$$\begin{aligned} \Pi_{\alpha\beta} = \eta^{\sigma\rho} (H^\mu{}_{\sigma\alpha} S_{\mu\rho\beta} - H^\mu{}_{\sigma\beta} S_{\mu\rho\alpha} - \\ H^\mu{}_{\alpha\sigma} S_{\mu\rho\beta} + H^\mu{}_{\alpha\sigma} S_{\mu\beta\rho}). \end{aligned} \quad (42)$$

There are nonlinearities in the field equations 29-30 and the continuity equations because gravitons have spin, polarization and momentum and couple to themselves.

When  $H_{\mu\nu\lambda} = 0$ , the field equations without an angular momentum source revert to the Abelian equation,

$$\partial^\mu (\partial_\mu G_{\nu\lambda} - \partial_\nu G_{\mu\lambda}) = 8\pi T_{\nu\lambda}, \quad (43)$$

which, under the harmonic gauge condition, is,

$$\square G_{\mu\nu} = 8\pi T_{\mu\nu}. \quad (44)$$

From this equation, the vortex model, and the geodesic equation derived in the next section, I will show that the theory agrees with all observations of N-body motion.

### E. Equation of test particle motion

First write the SO(4,1) Yang-Mills matrix potential in component form:  $A_\mu = G_{\mu\nu} V^\nu + H_{\mu\nu\lambda} M^{\nu\lambda}$ . The test particle has no internal structure and is represented simply as a parameterized path  $x^\mu(\tau)$ . Given the basic structure of the theory from Section II B, the potential,  $G_{\mu\nu}$ , takes the place of the metric normally found in Lagrangians for test particle motion.

Indexes are raised and lowered with the Minkowski metric  $\eta_{\mu\nu}$ . The Lagrangian for motion is:

$$\mathcal{L} = G_{\mu\nu} \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma} + H_{\mu\nu\lambda} \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma} \frac{dx^\lambda}{d\sigma}. \quad (45)$$

This describes both the curvature and twisting of the path as it moves through spacetime along its shortest path. (One can derive this equation by looking at the discrete lattice in Section II B and describing a path as a sequence of connected edges in the lattice.)

No observations of gravitational phenomena, save perhaps cosmological observations, are sensitive enough to measure torsion. Therefore, we concern ourselves in this section only with geodesic motion. Let  $H_{\mu\nu\lambda} = 0$ . (A similar assumption is made in applying Einstein-Cartan theory.) Let the Lagrangian for particle motion be,

$$\mathcal{L} = G_{\mu\nu} \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma}. \quad (46)$$

The equations of motion are given by the Euler-Lagrange equation,

$$\frac{d}{d\sigma} \left( \frac{\partial \mathcal{L}}{\partial (dx_\nu/d\sigma)} \right) - \frac{\partial \mathcal{L}}{\partial x_\nu} = 0. \quad (47)$$

The equation of motion for the particle (the “geodesic”) is,

$$\frac{d^2 x^\lambda}{d\tau^2} + \frac{1}{2} (G^{-1})^{\lambda\nu} [\partial_\rho G_{\mu\nu} + \partial_\mu G_{\rho\nu} - \partial_\nu G_{\mu\rho}] \frac{dx^\mu}{d\tau} \frac{dx^\rho}{d\tau} = 0, \quad (48)$$

which is the geodesic equation. Note: there is a distinction between raising indexes and inverting  $G_{\mu\nu}$ . Its inverse is  $(G^{-1})^{\mu\nu}$  such that  $(G^{-1})^{\mu\alpha} G_{\alpha\nu} = \delta_\nu^\mu$ , not  $G^{\mu\nu} = \eta^{\mu\alpha} \eta^{\nu\beta} G_{\alpha\beta}$ .

## F. Strong Equivalence Principle

Equivalence principles are defining features of gravitational theories. Newton’s theory included the Weak Equivalence Principle (WEP) where an object’s weight was assumed to be proportional to its mass. Einstein then developed the equivalence principle named for him, the Einstein Equivalence Principle (EEP), which states that local experiments in free fall are independent of position and velocity. When strongly gravitating bodies are taken into account, the validity of the equivalence principles come into doubt however as internal structure may cause violations of WEP such as the (never observed) Nordtvedt effect [11].

The Strong Equivalence Principle (SEP) is a natural extension of early equivalence principles. It has three conditions: (a) an object’s weight is proportional to its mass (WEP)

for self-gravitating bodies as well as test bodies, (b) the outcome of any local experiment is independent of the velocity of the (freely falling) apparatus, and (c) the outcome of any local experiment is independent of where and when in the universe it is performed [11].

Although a rigorous proof has not been found, GR is the only known metric theory that strictly satisfies SEP. Because experiment has never found a violation of SEP, any new gravitational theory must satisfy strict constraints on SEP violations. In the following I show that the Yang-Mills theory satisfies SEP strictly (1) because it is not a metric theory and its artificial geometry is not affected by gauge transformations and (2) because its gauge symmetry allows the required transformations that satisfy SEP as described in the next paragraph.

By satisfying all three conditions, the Yang-Mills theory satisfies SEP. The Yang-Mills theory satisfies the first condition, (a), by the arguments of Section 20.6 of [7] also found in [21]. If the field of a self-gravitating body asymptotically approaches “flatness” ( $G_{\mu\nu}(R) = 0$ ) at some distance,  $R$ , considered to be the boundary of the local system, this is sufficient to guarantee that a body’s self-gravitation and other internal structure does not affect its motion. Because it is always possible to find a gauge that eliminates the gravitational field at the boundary between a local, spherically symmetric, compact system and the external environment, a spherically symmetric self-gravitating body, even a neutron star or black hole, can be regarded as a point particle. In bimetric theories, which are diffeomorphism-covariant, changing the gauge to eliminate the field at the boundary changes the Minkowski metric, so a coordinate system cannot be found that satisfies this requirement. With the Yang-Mills theory, however, which also incorporates the Minkowski metric, the gauge can be changed without changing the coordinate system. The Minkowski metric field  $\eta_{\mu\nu}$  that gives distances in the artificial coordinate system does not change when the gauge of the gravitational field  $G_{\mu\nu}$  changes. Therefore, the Yang-Mills theory satisfies condition (a).

The Yang-Mills theory also satisfies the second condition, (b), by local Lorentz covariance (a subset of local de Sitter covariance) which means that experimental outcomes are independent (by gauge covariance) of Lorentz boosts and rotations and satisfies (c) by having no preferred location/time in the field equations. Unlike Rosen’s bimetric theory which has a local gravitational constant that depends on the field, the Yang-Mills theory has a non-location specific gravitational constant,  $a_g$ . Thus, all three conditions are met, and the Yang-Mills theory satisfies SEP.

## G. Quantization

The quantization of the theory is given by the generating functional:

$$Z[T, S] = \int DGDH \exp \left[ -\frac{i}{4} \int d^4x E_{\mu\nu\lambda} E^{\mu\nu\lambda} + F_{\mu\nu\alpha\beta} F^{\mu\nu\alpha\beta} + i a_g^2 \int d^4x G_{\mu\nu} T^{\mu\nu} + H_{\mu\nu\lambda} S^{\mu\nu\lambda} \right] \quad (49)$$

Because it is a massless gauge boson Yang-Mills theory, the theory given in this paper has a finite polynomial, positive definite action (20). Since generic Yang-Mills theories on semi-simple groups are renormalizable [5], this one is as well. This constitutes mathematical proof that the SO(4,1) Yang-Mills theory in this paper is renormalizable.

The standard quantization scheme is appropriate with Fadeev-Popov ghost fields used to derive Feynman rules including the graviton propagator,

$$D_{\mu\nu}^{\lambda\rho}(p) = \frac{-i\delta^{\lambda\rho}}{p^2 + i\epsilon} \left[ \eta_{\mu\nu} - \frac{(1 - \xi)p_\mu p_\nu}{p^2 + i\epsilon} \right], \quad (50)$$

where  $\lambda, \rho$  are group indexes [5].

## III. EXPERIMENTS AND OBSERVATIONS

Numerous observations, starting with light bending in 1919, have been made to attempt to confirm predictions of general relativity. These include gravitational time dilation, redshift, and light bending which have been measured within the solar system as well as perihelion precession of the planets and binary pulsar precession [6]. Other observations do not agree with the original theory and have required modifications to the Einstein field equations. In order to show that the theory agrees with as many observations as possible, in the following, I derive (1) a static, spherically symmetric solution to the field equations, (2) the 1PN equations of motion for a binary system, (3) the radiation reaction for binary pulsar inspiral, and (4) predict the expansion of the universe via a homogeneous, isotropic model.

### A. Spherically symmetric solution

The static, spherically symmetric Schwarzschild solution is one of the most important solutions to the Einstein field equations. The solution to the de Sitter abelian field equations

44 is identical up to linear order. Let spacetime be covered by spherical coordinates  $(t, r, \phi, \theta)$  where  $\theta$  is colatitude and let the metric be the flat spacetime metric [6],

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2. \quad (51)$$

The only allowable non-zero components of the potentials are  $G_{00}$  and  $G_{rr}$ . Choosing a harmonic gauge we have,

$$\nabla^2 G_{00} = 8\pi T_{00}, \quad (52)$$

$$\nabla^2 G_{rr} = 8\pi T_{rr}. \quad (53)$$

Placing the source at the origin, in spherical coordinates, its tensor has components  $T_{00} = T_{rr} = \rho$  such that its density is  $\rho = M\delta(r)$ . Recalling the flat space boundary conditions that  $G_{\mu\nu} \rightarrow \eta_{\mu\nu}$  as  $r \rightarrow \infty$ , the solutions to the field equations are:

$$G_{00} = -\left(1 - \frac{2M}{r}\right), \quad G_{rr} = 1 + \frac{2M}{r}, \quad G_{\theta\theta} = r^2, \quad G_{\phi\phi} = r^2 \sin^2(\theta). \quad (54)$$

This is the linearized Schwarzschild solution to the Einstein equations [6] but, for this Yang-Mills field theory, it is exact for all spherically symmetric bodies, including black holes.

## B. Particle trajectories

Particle trajectories in a static, spherically symmetric potential field are identical to those of general relativity. The only difference is in the radial potential. The Schwarzschild radial potential  $g_{rr} = (1 - 2M/r)^{-1}$  differs from the radial potential derived in Section III A to quadratic order,

$$G_{rr} = 1 + 2M/r \approx (1 - 2M/r)^{-1} + O((M/r)^2). \quad (55)$$

No tests within the Solar System, whether with respect to the Sun, Earth, or another body (the most precise tests have been done near the Earth) have achieved better than linear order in Schwarzschild coordinates [7][6] because of the weak fields involved. If  $M = Gm/c^2$  is the Schwarzschild radius and  $m$  is the mass in other units (e.g., kilograms) at the surface of the Sun,  $2M_{\odot}/r \approx 4.25 \times 10^{-6}$  and at the surface of the Earth  $2M_{\oplus}/r \approx 1.4 \times 10^{-9}$ . Both are much smaller than necessary to measure higher order effects where imperfections in density (e.g., mountains) would make measurements difficult to verify.

Predictions of gravitational time dilation and redshift, meanwhile, are mathematically identical in both theories (to any order) given a static, spherically symmetric field with redshift proportional to  $1 + 2M/r$ . Recent high precision measurements of redshift using cesium atoms in a laboratory agree with predictions of both theories [22]. Therefore, no tests done within the Solar system to date disprove either theory. For strong field tests, we look outside the solar system to binary pulsars and, further on, cosmology.

### C. Post-Newtonian Equations of Motion

The binary pulsar system B1913+16 discovered in 1974 provided one of the first tests of strong field general relativity [21]. Because of the regular radio pulses of the star which is orbiting a relatively inert body such as a dead pulsar, henceforth called the companion body, allowed extremely precise measurements of the pulsar's orbit, the small deviations of the orbit from Kepler's laws can be measured, including orbital damping caused by gravitational radiation, which is a higher order effect than orbital precession [21]. Observation of this system over the past thirty-five years have only increased the accuracy of the measurements. Recently, the double binary pulsar system PSR J0737-3039A/B with two active pulsars orbiting each other has been observed providing even greater confirmation of GR's predictions [8]. The following demonstrates that the field equations of Sec. II C also predict the observations of these pulsar systems.

Although it is not a metric theory, its close resemblance to a metric allows the G-field to be expressed in a form similar to PPN form. Therefore, in this section, I apply this framework to show that the many-body equations of motion agree with those of general relativity up to first post-Newtonian corrections. The parameterized post-Newtonian (PPN) equations of motion of gravity are the primary vehicle by which not only different theories of gravity (including GR) are compared but provide the equations for predicting the motion of two or more gravitating bodies. Tests of these equations include the measurement of planetary motion within the Solar system for weak fields and binary pulsar precession and gravitational radiation for stronger fields [7] [11].

For a binary pulsar system the relevant metric in the post-Newtonian coordinate system

for general relativity is ([11], 11.52)

$$g_{00} = -1 + 2 \sum_{a=1,2} m_a / |\vec{x} - \vec{x}_a(t)| + O(\epsilon^4), \quad (56)$$

$$g_{0j} = O(\epsilon^3), \quad (57)$$

$$g_{ij} = \delta_{ij} \left( 1 + 2 \sum_{a=1,2} m_a / |\vec{x} - \vec{x}_a(t)| \right) + O(\epsilon^4), \quad (58)$$

with  $m_1$  and  $m_2$  being the masses of the bodies and  $\vec{x}_1$  and  $\vec{x}_2$  their positions in the appropriate harmonic coordinate system. From the relaxed field equations (44),

$$G_{00} = -1 + 2 \sum_{a=1,2} \frac{m_a}{|\vec{x} - \vec{x}_a|} + 2\Psi + O(\epsilon^6), \quad (59)$$

where

$$\Psi(t, \vec{x}) = \sum_{a=1,2} \frac{m_a v_a^2}{|\vec{x} - \vec{x}_a|} + \sum_{a,b=1,2, a \neq b} \frac{m_a}{|\vec{x} - \vec{x}_a|} \frac{m_b}{|\vec{x}_a - \vec{x}_b|}, \quad (60)$$

$$G_{0j} = -2 \sum_{a=1,2} \frac{m_a v_{aj}}{|\vec{x} - \vec{x}_a|} + O(\epsilon^5), \quad (61)$$

and

$$G_{ij} = \delta_{ij} \left( 1 + 2 \sum_{a=1,2} \frac{m_a}{|\vec{x} - \vec{x}_a|} \right) + O(\epsilon^4). \quad (62)$$

The velocities of the bodies are  $\vec{v}_1 = (v_{11}, v_{12}, v_{13})$  and  $\vec{v}_2 = (v_{21}, v_{22}, v_{23})$  and  $v_a = \|\vec{v}_a\|$ . (See Appendix B for a derivation of these potentials.)

Dropping the extra terms from  $G_{\mu\nu}$  for this system that exceed the required order, the 1PN metric matches  $G_{\mu\nu}$ . Since the theory satisfies SEP, for any set of compact nearly-spherical bodies where tidal forces may be neglected, the geodesic equation 48 predicts orbital motion. The post-Keplerian parameters of periastron advance,  $\langle \dot{\omega} \rangle$ , time delay,  $\gamma'$ , and Shapiro delay parameters,  $r$  and  $s$ , are also, consequently, the same in both theories.

The final post-Keplerian parameter, orbital speed-up caused by gravitational radiation,  $\dot{P}_b$ , derives from the quadrupole formula from the Yang-Mills field equations discussed in the following section.

### 1. Orbital speed-up of a binary pulsar system

Gravitational radiation was first addressed by Einstein shortly after the publication of general relativity, where he demonstrated that the radiation is primarily quadrupolar in

contrast to the primarily dipolar radiation from electromagnetic sources [23]. The energy loss of a binary system of compact stars was first demonstrated in a paper by Peters and Matthews [24], who derived the energy loss of binary stars in Keplerian orbit leading to the formula for the orbital speed-up,

$$\dot{P}_b = -\frac{192\pi G^{5/3}}{5c^5} \left(\frac{P_b}{2\pi}\right)^{-5/3} (1-e^2)^{-7/2} \times \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) m_p m_c (m_p + m_c)^{-1/3}, \quad (63)$$

with  $m_p$  and  $m_c$  the masses of the pulsar and companion body respectively and  $e$  the orbital eccentricity [25]. Although this equation was developed for Keplerian orbits, it has been applied to post-Keplerian orbits since the only requirement is that the orbit be elliptical, and, although much higher order derivations have been made [11], only this equation has been tested. In this section, I demonstrate an identical increase in orbital speed for the Yang-Mills theory via the same techniques but omit a general discussion of gravitational waves. A detailed discussion of multipole expansions for gravitational radiation in general relativity can be found in [26].

Orbital speed-up is a function of energy loss, i.e. radiation-reaction, which to lowest multipole order in General Relativity is ([7], 36.31),

$$\frac{dE}{dt} = -\frac{1}{5} \left\langle \frac{d^3 Q_{jk}}{dt^3} \frac{d^3 Q_{jk}}{dt^3} \right\rangle, \quad (64)$$

where  $Q_{\mu\nu}$  is the reduced quadrupole moment (the trace free part of the second moment of the mass distribution) such that  $Q_{\mu\nu} = q_{\mu\nu} - \frac{1}{3}\delta_{\mu\nu}q$ ,

$$q_{\mu\nu} = \int d^3x \rho_0 x_\mu x_\nu, \quad (65)$$

and  $q = q_\mu{}^\mu$ . The Peters-Matthews formula for energy loss of a binary system ([11], 10.80),

$$\frac{dE}{dt} = -\left\langle \frac{\mu^2 m^2}{r^4} \frac{8}{15} (12v^2 - 11\dot{r}^2) \right\rangle, \quad (66)$$

where  $m = m_1 + m_2$ ,  $\mu = m_1 m_2 / m$ ,  $v = |\vec{v}_1 - \vec{v}_2|$ , and  $r = |\vec{x}_1 - \vec{x}_2|$ , can be found from the linearized vacuum equations of general relativity,  $\square \bar{h}^{\mu\nu} = 0$ , [24][7] by the quadrupole relation,

$$\bar{h}_{jk}(t, \vec{x}) = \frac{2}{r} \ddot{q}_{jk}(t - r), \quad (67)$$

for the spatial part of the radiation field ([7], 36.50) where  $r$  is the distance from the source.



To show that both GR and the Yang-Mills theory have the same orbital speed-up, we compare the equations for the two theories. Peters and Matthews derives the energy loss using the linearized Einstein vacuum equations:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (68)$$

such that

$$\square \bar{h}_{\mu\nu} = 0, \quad (69)$$

where  $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h_\lambda{}^\lambda$ .

The linearized Yang-Mills equation for an unpolarized, spinless source,

$$\square G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad (70)$$

is the same as the linearized equation for GR.

Let  $G_{\mu\nu} - \eta_{\mu\nu} = h'_{\mu\nu}$ . A plane wave solution to the wave equation,

$$\square h'_{\mu\nu} = 0, \quad (71)$$

is

$$h'_{\mu\nu} = ae_{\mu\nu} \cos(\omega t - \vec{k} \cdot \vec{x}), \quad (72)$$

where  $a$  is the amplitude,  $e_{\mu\nu}$  is a symmetric, traceless, transverse, and unitary polarization tensor (as defined in [24]), and  $\omega$  is the frequency.

In the following, I derive the quadrupole expansion for plane wave solutions to the linearized Yang-Mills (YM) theory. An important difference between electromagnetism and gravity is that, whereas the primary electromagnetic radiation for charged bodies in motion is dipole, the primary gravitational radiation for massive bodies in motion is quadrupole. This derivation follows the outline in [11], Chapter 10.

The multipole expansion of the plane wave solution (72) is,

$$h'_{\mu\nu} = 4r^{-1} \sum_{m=0}^{\infty} (1/m!) (\partial/\partial t)^m \int d^3x' T_{\mu\nu}(t-r, \vec{x}') (\vec{n} \cdot \vec{x}')^m. \quad (73)$$

to 1PN order, where  $\vec{n} = \vec{x}/r$ . Because of the gauge condition,  $\partial_\mu G^{\mu\nu} = 0$ , and the retarded potential, we can use the following relations,

$$\partial_0 h'_{0k} = n^j \partial_0 h'_{jk}, \quad \partial_0 h'_{00} = n^j n^k \partial_0 h'_{jk}, \quad (74)$$

and only need to determine the  $h'_{jk}$  components [11]. Because, to order,  $\partial_\mu T^{\mu\nu} = 0$  by conservation and that the source is symmetric, a useful relation is,

$$(\partial^2/\partial t^2) \int d^3x T^{00} x^j x^k = 2 \int d^3x T^{jk} \quad (75)$$

Therefore, 73 becomes,

$$h'_{ij} = 2r^{-1}(\partial^2/\partial t^2) \left( \int d^3x T^{00}(t-r, \vec{x}) x_i x_j \right) + \text{higher order}. \quad (76)$$

This means that monopole and dipole moments of  $T^{ij}$  may be expressed as time derivatives of quadrupole moments of  $T^{00}$ . To post-Newtonian order  $T^{00} = \rho$ , the mass density, and we now have the quadrupole approximation.

For the binary system the integral simplifies to,

$$h'_{ij} = \frac{2}{r} \ddot{q}_{jk}(t-r). \quad (77)$$

Since from 67 we have  $h'_{ij} = \bar{h}_{ij}$  and because  $\bar{h}_{ij}$  solves the vacuum equation we also have  $\bar{h}_{ij} = h_{ij}$ . The speed-up formula (63) follows the same procedure as that of general relativity: take a Taylor expansion of 77 in powers of  $r$  to extract the radiation-reaction potential and applying the geodesic equation 48, determine the reaction acceleration of the bodies ([7], Sec. 36.11). Then arrive at 63.

In this section, I showed that the inspiral of orbiting binary pulsars is the same for both general relativity and the Yang-Mills theory. Because the most accurate measurements to date match 63, they agree with both theories.

## D. Cosmology

If this paper had been written as recently as 10-15 years ago, cosmology could have been addressed more fully within the scope of this paper. In recent years, however, the amount of data available to match to cosmological models has exploded thanks to projects such as WMAP [4], observations of Type 1a supernovas, and baryon acoustic oscillation measurements. In this section, I briefly address the Robertson-Walker model, the starting point of the two most prominent numerical approaches, both centered around the Friedmann equations: perturbations of the Einstein field equations and N-body cosmology which models the universe as an N-body Newtonian system.

The Friedmann equations can be derived from Newton's laws, suggesting that at slow speeds and very weak fields they are valid for the universe as a whole—certainly the present universe is essentially Newtonian at large scales where inhomogeneities are small. In the early universe, however, relativistic speeds and strong gravitational fields predominated and the Newtonian model breaks down for times “close” to the Big Bang.

### 1. *Isotropic, Homogeneous Universe*

Observations of the cosmic microwave background and statistical counts of the distribution of galaxies imply that the universe is statistically isotropic [6]. By the Copernican principle that we do not occupy a privileged location in the universe, the universe must also be homogeneous. An isotropic, homogeneous universe has a very simple dynamical description. For a perfect fluid universe, the stress-energy-momentum tensor in special relativity has the standard form,

$$T^{\mu\nu} = \rho u^\mu u^\nu + p(u^\mu u^\nu + g^{\mu\nu}), \quad (78)$$

where  $\rho$  is mass density and  $p$  is pressure [6]. The fundamental Friedmann equations for general relativity are,

$$\dot{a}^2 + k = \frac{8\pi\rho a^2}{3}, \quad (79)$$

$$\frac{3\ddot{a}}{a} = -8\pi(\rho + 3p), \quad (80)$$

where  $k$  is the curvature constant [10] and

$$g_{00} = -1, \quad g_{rr} = \frac{a^2(\tau)}{1 - kr^2}, \quad g_{\theta\theta} = a^2(\tau)r^2, \quad g_{\phi\phi} = a^2(\tau)r^2 \sin^2 \theta, \quad (81)$$

is the metric. Recent observations that the universe's density, the sum of baryon, dark matter, and dark energy densities, is near or at the critical value,  $\Omega_b + \Omega_c + \Omega_\Lambda \simeq \Omega_{crit}$ , and the universe is spatially flat,  $k \approx 0$  [4][9]. Therefore, for the rest of this section,  $k = 0$ . From the conservation of energy,

$$\frac{\partial \rho}{\partial \tau} + \frac{3\dot{a}}{a}(\rho + p) = 0, \quad (82)$$

is a fundamental equation for the evolution of matter and energy.

Turning to the Yang-Mills theory, I derive the equations of motion for the universe. Modern cosmology relies heavily on perturbation methods, and the Robertson-Walker assumptions of isotropy and homogeneity are starting points for this approach because they

generate the unperturbed solutions to the field equations. Therefore, I again make these assumptions.

The isotropic, homogeneous universe under the General Relativity model does not distinguish between matter moving apart within space and space itself moving apart. Hence, redshifts are interpreted as being caused by matter moving apart as space drags it along in its expansion. It is certainly possible, however, to distinguish between the two types of redshift, gravitational and doppler, in ordinary, non-cosmological circumstances such as within the Solar System. Making such a distinction is important when attempting to measure (1) the age of the universe, which is based on redshift measurements, and (2) whether matter is accelerating away or experiencing increasing gravitational redshift and moving away at fixed velocity. These measurements, in turn, affect how observations such as abundances of light elements are interpreted [16].

In this section, I show how the theory supports a linear coasting or Milne cosmology with some modifications: In the Poincaré approximation, gravitational redshift is linear. Thus, in that approximation all observed redshift is caused by matter flying apart. In the de Sitter theory, however, the influence of the torsion potential,  $H$ , causes gravitational redshift to increase causing redshift to accelerate with time and giving the impression of an accelerating expansion.

As discussed in section II B, the coordinate system is a set of degrees of freedom, that, while having no physical meaning, may be used to simplify equations. In this cosmology, I will use comoving coordinates again,  $g_{00} = -1$ ,  $g_{ij} = a^2(t)\delta_{ij}$ , to expand with matter moving *within* space and define the expansion of space itself with the following,  $G_{00} = -1$  and  $G_{ij} = b(t)a^2(t)\delta_{ij}$ . The gauge is also comoving. As we will see, the torsion potential is non-zero,  $H_{\mu\nu\lambda} \neq 0$ , because where two  $G$  gravitons couple they produce  $H$  gravitons. This is a consequence of the de Sitter Lie algebra  $[V_\mu, V_\nu] \neq 0$  and disappears in the Poincaré approximation where  $[V_\mu, V_\nu] = 0$  (see B). Let  $H_{i0i} = -H_{ii0} = c(t)a^2(t)$  and  $H_{ijj} = -H_{jji} = d(t)a^2(t)$  be the isotropic, homogeneous torsion potential.

In the comoving coordinates, the conserved current is  $T_{00} = \rho$  and  $T_{ij} = pa^2\delta_{ij}$ . The spin density is zero by isotropy,  $S_{\mu\nu\lambda} = 0$ . Evaluating the field equations 29, we have,

$$\frac{3\dot{a}(a\dot{b} + b\dot{a} - \dot{a} + ca)}{a^2} = 8\pi\rho, \quad (83)$$

and the next three (all the same) are,

$$\frac{-3\dot{a}\dot{b}a - \ddot{b}a^2 - \ddot{a}ab - \dot{a}^2b + a\ddot{a} - \ddot{b}a^2 - \dot{c}a^2 - 2\dot{a}ac}{a^2} = 8\pi p. \quad (84)$$

From the field equations 30, we have,

$$\ddot{c}a^2 + 3\dot{c}\dot{a}a - 3\dot{a}^2c + \ddot{a}ac - \dot{a}ab + a\ddot{a} - \dot{b}a^2 - ca^2 = 0 \quad (85)$$

and

$$\ddot{d}a^2 - \dot{a}^2d + a\dot{a}\dot{d} - a^2b^2d = 0. \quad (86)$$

By the vortex model all matter has  $p = \rho/3$  which simplifies 40 to

$$\dot{\rho} = -4\frac{\dot{a}}{a}\rho \quad (87)$$

The system of three differential equations (83,84, and 87) govern the gravitational behavior of matter in this universe. The boundary condition are given in terms of present day,  $t_0$ , and origin,  $t = 0$ , parameters:  $a(t_0) = a_0$ ,  $a(0) = 0$ ,  $b(0) = 0$ ,  $c(0) = 0$ , and  $c(t_0) = c_0$ . The assumption of  $a(0) = 0$  is a classical assumption. Quantum effects take over at  $t \ll 1$  and may predict a different initial size of the universe. Certainly, the matter in the universe could never be localized to a point of zero radius as this would violate the uncertainty principle.

The equations 83,84, and 87 have the non-trivial solution,

$$a(t) = \frac{a_0}{t_0}t, \quad (88)$$

$$\rho(t) = \rho_0 \frac{a_0^4}{a^4}, \quad (89)$$

$$b(t) = \frac{1}{12t^2} [Ct^4 - 3Ct^3 + (12 + 8\pi\rho_0t_0^4)t^2 - 40\pi\rho_0t_0^4t - 32\pi\rho_0t_0^4], \quad (90)$$

$$c(t) = \frac{1}{12} [-3Ct + 8\pi\rho_0t_0^4], \quad (91)$$

where  $C = \frac{1}{3t_0} (-12c_0 + 8\pi\rho_0t_0^4)$ . The equation for  $d(t)$  is not expressible in closed form but is the solution to a second order ODE.

The rate of expansion of the universe,  $v$ , is related to the Hubble constant by,

$$v \equiv \frac{dR}{d\tau} = HR, \quad (92)$$

where  $R$  is the distance between two isotropic observers [6], and

$$H = \frac{\dot{a}}{a} = 1/t, \quad (93)$$

where currently accepted value of the Hubble constant is  $70.1 \pm 1.3 (km/s)/Mpc$  [4]. This is the apparent Hubble constant, however. Let  $\beta(t) = \sqrt{b(t)}$ . Then the apparent Hubble constant is:

$$\tilde{H} = \frac{\dot{\beta}a + \beta\dot{a}}{\beta a} = \frac{\dot{\beta}}{\beta} + H. \quad (94)$$

This implies that the universe's age should be given by,

$$t_0 = \frac{1}{\tilde{H} - \frac{\dot{\beta}}{\beta}}. \quad (95)$$

If  $\frac{\dot{\beta}}{\beta} > 0$  the universe is older than it appears. If  $\frac{\dot{\beta}}{\beta} < 0$  it is younger than it appears. Given that the redshift is accelerating, the former is true, and the universe is older than it appears. This makes sense because gravitational redshift on top of Doppler redshift would make the expansion of the universe appear faster than it is implying that its origin was sooner in the past than the Doppler redshift alone would imply.

The scale factor,  $a(t)$ , conforms to a universe with a Minkowski metric sometimes called the Milne universe or simply the linear model [27]. The Milne cosmological model has recently seen a resurgence as an alternative to the standard  $\Lambda$ CDM model and has some compelling features: It gives the same age for the universe as the  $\Lambda$ CDM model [28]. Nucleosynthesis has been fit to observations for the linear model in [29] with some problems that I discuss below [16]. Predictions for Type 1a supernovae are very close to those of the standard model. The Milne universe has angular distances in the CMB of about 1.1 degrees, also close to the observed value [15] [30].

A variation known as the “symmetric” Milne model makes use of equal amounts of matter and antimatter present in the universe to account for zero energy density, assuming that antimatter has negative mass [15]. This assumption is not necessary here. Indeed, antimatter in the Yang-Mills theory has positive mass and energy (Section II A).

Although the rate of expansion is linear, the gravitational potential,  $b(t)$ , shows an acceleration in apparent expansion rate, related to  $\sqrt{G_{ij}\delta^{ij}}/3 = \sqrt{b(t)a^2(t)} \sim O(t^2)$ , meaning that the redshift of distant galaxies should be accelerating linearly with time even while the galaxies are actually moving away at a constant rate given by  $a(t)$ . At the present time, the acceleration is not well understood enough to determine whether it is linear, exponential (as in models with a cosmological constant), or some other function of time. Hence, should better measurements become available that show the rate of acceleration, they may test the theory's predictions.

## 2. The Poincaré approximation

When the Poincaré approximation is made such that we change the equations to conform to the Lie algebra with  $[V^\mu, V^\nu] \approx 0$  (see B), the expansion no longer accelerates:

$$a(t) = a_0 t / t_0, \quad (96)$$

$$\rho(t) = \rho_0 \left( \frac{a_0}{a} \right)^4, \quad (97)$$

$$b(t) = 1 - \frac{8\pi\rho_0}{3(a_0^4/t_0^4)t^2} + \frac{K}{(a_0/t_0)t}, \quad (98)$$

where  $K = (b_0 + 8\pi\rho_0/(3a_0^4/t_0^2) - 1)a_0$  and  $b_0 = b(t_0)$ . Note how  $b(t) \rightarrow 1$  as  $t \rightarrow \infty$  indicating that the Poincaré universe has no accelerating expansion. The explanation is that when  $[V^\mu, V^\nu] = iM^{\mu\nu}$  (see A), it means that momentum, related to the  $V^\mu$  generators, produces spin. Because  $[M_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho}V_\nu - \eta_{\nu\rho}V_\mu)$  spin gravitons couple to momentum gravitons to create more momentum gravitons. Since the potential for momentum gravitons is  $G_{\mu\nu}$  this relationship causes the redshift we see. The result is an accelerating expansion of gravitational redshift.

Another physical phenomenon has a similar explanation: Thomas Precession. In that case, the Lorentz rather than de Sitter algebra works to relate orbital momentum of the electron to changes in its spin [14]. Thus, “dark energy” can be explained as the production of gravitons by an interplay between spin and momentum.

## IV. POTENTIAL DISAGREEMENTS

Observations of gravity can be classed into a four basic types: (1) Newtonian gravity which applies to very weak fields and slow velocities that pertain to most galaxies, stars, and orbits of the outer planets, (2) weak field gravity responsible for classic effects such as redshift, light bending, time dilation, perihelion precession (3) strong field gravity responsible for binary pulsar precession, orbital damping, and gravitational waves, and (4) cosmology where assumptions such as homogeneity and isotropy greatly simplify equations as well as perturbative cosmology (not addressed in this paper). Included are tests of the Strong Equivalence Principle (SEP).

Going through each of the four types of tests:

*Newtonian gravity.* Both theories subsume Newtonian gravity at very weak field strengths and slow velocities, and this is easily demonstrated from the field equations.

*Weak Fields.* As shown in section III B, both theories agree with classic effects up to linear order in the Schwarzschild radius over the distance. Measurements of redshift, light bending, perihelion precession, and time dilation have not been made to quadratic accuracy in Schwarzschild coordinates (as opposed to isotropic coordinates where the coordinate change introduces an artificial “measurable” quadratic term); therefore, none of these experiments contradict the YM theory.

*Strong Fields.* As [8] mentions, measurements of binary pulsar precession, while some of the most precise measurements of relativistic gravity ever made, are not sufficiently accurate to confirm beyond the 1PN equations of motion. Orbital damping caused by radiation reaction is a higher order effect but also not confirmed beyond the quadrupole approximation, and neither GR nor the YM theory predict a dipole moment. Therefore, as shown in section III C, none of these measurements contradicts either theory nor any other measurements of multibody systems. The YM theory’s geodesic equation 48 is also the same as in relativity; therefore, since SEP is satisfied (Sec. II F) and compact self-gravitating bodies behave as if they are test bodies, it agrees with the 1PN equations of motion of general relativity. Gravitational wave measurements promise to provide higher order estimates which may show a violation of one of the theories, but these, as yet, are not available.

*Cosmology.* The highest order measurements available are cosmological. In this case, several sources including Type Ia supernovae (SN), baryon acoustic oscillation (BAO), and the cosmic microwave background (CMB) provide data that can be used to constrain various theories [9]. The model we have presented is essentially a toy model because it assumes no fluctuations or deviations from perfect uniformity and isotropy in the universe. First order perturbation of the field equations about the Robertson-Walker solution combined with numerical solvers can indicate whether observed fluctuations in the CMB agree with the theory, and this is an important future direction of research for the YM theory.

Studies of Big Bang Nucleosynthesis (BBN) where the predicted quantities of light elements such as Helium, Lithium, and Deuterium are compared with predicted quantities can also constrain some theories and several studies of Tensor-Scalar theory have been done [31][32]. These studies tend to focus on the effects of the scalar field on nucleosynthesis and are not directly applicable to the YM theory which has no scalar field.



While the linear model explains features of the later universe as well as the standard  $\Lambda$ CDM model [15], a single power law scale factor has proved insufficient to explain the early universe where the power is sharply constrained to around  $a \sim t^{0.55}$  [16] unless additional modes of production of deuterium are invoked in the later universe. Because the YM theory predicts a gravitational redshift on top of the Doppler redshift, however, the age of the universe may be different than currently predicted. Since the age is an essential assumption of nucleosynthesis predictions, this throws the arguments against the linear model, in the context of the YM theory, into doubt. Whether the theory is able to predict abundances of light elements is left for future work.

## V. FUTURE TESTS

General Relativity and the Yang-Mills theory agree for the linearized Schwarzschild metric predictions and 1PN equations of motion but, with the exception of radiation-back-reaction where the same degree of inspiral is predicted for binary pulsar systems, they fundamentally diverge at higher order motion. The 2PN order should be measurable for binary pulsars within the next few decades as additional data is collected about known systems (the more data collected the smaller the error bound). Gravitational wave detectors such as LIGO and VIRGO may also be capable of detecting much higher order effects (well into the quadratic or even cubic realm) from inspiraling black holes and other closely spaced, highly massive objects. Because general relativity and the Yang-Mills theory no longer agree at this level, these observations will determine whether gravity is truly diffeomorphic or a non-Abelian gauge theory like the other known forces. Another important test is a quantitative measurement of the values of  $b_0$  and  $c_0$  which may allow a quantitative prediction of the rate of acceleration of the universe, an important prediction of the theory.

Quantum predictions are significant because the theory, in order to be renormalizable, requires mass, energy, momentum, and angular momentum to be quantized so that the coupling constant has zero mass dimension. Without that assumption, the coupling constant would have negative mass dimension of -2 and the theory would not be renormalizable [5]. The size of the quantum must be exceptionally small, but an experiment may be devised to detect it similar to how Millikan's oil drop experiment detected the indivisibility of the electron charge.

## VI. RELATED WORK

While this paper has focused on the problems with the prevailing theory of gravity, general relativity, several other theories have been proposed to solve quantum gravity and explain macroscopic observations. The most important empirical theory is the Lambda-CDM model which was used in the WMAP survey [4] and combines the Einstein field equations with a cosmological constant and cold dark matter. The field equations 29-30 do away with the cosmological constant and much of the complexity of the curved spacetime model, but the YM theory does not challenge the prevailing theory of dark matter.

The most prominent theory of quantum gravity is string theory and its derivative M-theories which, rather than being a simple theory of gravity, is an extensive modification of physical law positing that all matter is composed of strings, attempting to unify all forces [5][33]. Although unification is not its purpose, in representing gravity identically to the other three forces, the theory may possibly be unified with them at a high enough symmetry group without requiring any significant additional physical assumptions such as strings or additional spacetime dimensions.

The next most significant model of quantum gravity is loop quantum gravity [33]. Loop quantum gravity has a different approach from that given in this paper, further marrying gravity to curved spacetime. In making the assumption that gravity has only diffeomorphism covariance and is dominated by a metric geometry, it retains the complexity of general relativity and adds to it by introducing spin foams, i.e. discretizations of spacetime, in order to avoid blow-ups of the quantum variables. It also makes no predictions that are currently testable and, furthermore, has not been shown to agree with classical general relativity in its entirety.

The main Yang-Mills approach to gravity is conformal gravity [12], and is not directly relevant to the theory in this paper which makes significantly different physical and mathematical assumptions. Conformal gravity is a four derivative Lagrangian theory and predicts significantly different large scale behavior than either general relativity or Newtonian gravity.

## VII. CONCLUSION

The assumption of curved spacetime, while elegant, is not a necessary component given current experimental evidence, and, therefore, we are motivated to consider alternatives to it in a theory of gravity. By assuming a de Sitter symmetry group, this theory not only agrees with the observations confirming general relativity's major predictions but also includes a renormalizable quantum theory of gravity and a definition of mass as relativistic energy. It does this by establishing gravity on the theoretical foundation of the Standard Model. Indeed, the Standard Model  $SU(3) \times SU(2) \times U(1)$  is conjectured to be part of a unified, covariant theory in  $SU(5)$  at some energy since  $SU(5)$  is the smallest group that can contain all three. With the addition of the de Sitter group, the Standard Model could be updated to be  $SO(4,1) \times SU(3) \times SU(2) \times U(1)$ . The smallest group to contain this would be  $SU(10)$  with 99 degrees of freedom. Another possibility would be  $SO(15)$  which has 105 degrees of freedom. Either of these might represent a unified field theory at high enough energy.

The theory, in addition, solves at least two problems with General Relativity:

1. Quantum renormalizability. General Relativity has a non-renormalizable action while the YM theory has a renormalizable action provided mass and momentum are quantized. Although such quantization has not been measured, the weakness of gravity suggests that it is extremely small, much smaller than the quantization of charge.
2. Vacuum energy. Although vacuum energy is the predominant explanation for the source of dark energy, the vacuum energy has a density approaching  $\hbar$  [5]. This density is the density the universe would have if Planck mass black holes were set adjacent to one another, event horizon to event horizon at every point in space. Clearly, this is not so. It has been suggested that something cancels the vacuum energy out, but that has not yet been found. It is reasonable to believe that the coupling of gravity to the vacuum energy, far from being an explanation for dark energy, is, along with the accelerating expansion, an unsolved anomaly of the mainstream theory. By not coupling to the vacuum energy in its quantum formalism, the YM theory does not have the same problem.

The most important prediction, and the reason why this paper is more than a theoretical exercise, is the prediction of the accelerating expansion of the universe from the de Sitter

group. Although only qualitative here, it has the potential to be a quantitative prediction with additional data to constrain the current conditions of the universe within the context of this theory, particularly values of  $b_0$  and  $c_0$ . Further confirmations may be obtained from detection of gravitational waves, observations of supermassive objects such as black holes, and additional cosmological measurements. Additional numerical studies are required to determine how Big Bang Nucleosynthesis observations fit the theory.

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- [35] In this paper, I use the words “polarization” and “oscillation” and “angular momentum” and “spin” interchangeably. It is, however, the sum of polarization and classical oscillation and the sum of spin and classical angular momentum that are conserved by Lorentz symmetry.

## Appendix A: The de Sitter Group Lie Algebra

In addition to rotations and boosts, all physical theories are translation invariant. This confirms the assumption that there is no “special” place in the universe. By Noether’s theorem, translation invariance causes momentum to be conserved (including energy). A translation by an amount  $v_\mu$  is achieved by a  $5 \times 5$  matrix,  $T = v^\mu V_\mu$  such that  $V_\mu = (V_t, V_i)$  are four generators. Given a 4-vector  $u_\mu$  if  $w = (u_\mu, 1)$ , then  $w' = Tw = (u_\mu + v_\mu, 1)$ . It is well known, however, that prior to the introduction of Lorentz covariance, mechanical theories such as Newtonian gravity were Galilean invariant,  $R^3 \otimes \text{SO}(2)$  rather than Lorentz invariant,  $\text{SO}(3,1)$ . A reasonable extension of the Poincaré group, then, is from  $R^{(3,1)} \otimes \text{SO}(3,1)$  to  $\text{SO}(4,1)$  assuming that the radius of curvature in the fifth dimension is large so that it appears to be Poincaré invariant for small rotations.

In the de Sitter group,  $x_4$  is a fourth spatial dimension and has ordinary rotations with respect to the other three spatial dimensions and boosts with respect to time. The anti-de Sitter group has  $x_5$  as a second time dimension. Let  $V_\mu$  be rotations/boosts in the  $x_\mu$ - $x_4$  plane.

The Poincaré group has the covariant Lie algebra,

$$[V_\mu, V_\nu] = 0 \tag{A1}$$

$$[M_{\mu\nu}, P_\rho] = i(\eta_{\mu\rho} V_\nu - \eta_{\nu\rho} V_\mu) \tag{A2}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\mu\rho} M_{\nu\sigma} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho}), \tag{A3}$$

for  $\eta_{\mu\nu}$  the Minkowski metric where  $U = \exp[-\frac{i}{2}\omega_{\mu\nu} M^{\mu\nu}]$  is a Lorentz operation and  $U =$

$\exp[ia_\mu V^\mu]$  is the translation operation [34]. The only alteration that the de Sitter group makes is that the “translation” operators now generate a Lorentz rotation or boost,

$$[V_\mu, V_\nu] = iM_{\mu\nu}, \quad (\text{A4})$$

in which case

$$U = \exp\left[-\frac{i}{2}\omega_{\mu\nu}M^{\mu\nu} + ia_\mu V^\mu\right],$$

is the de Sitter operation.

## Appendix B: Derivation of the 1PN potential

The potential at a spacetime point  $(t, \vec{x})$  in the given inertial frame can be solved by the integral equations [21],  $h_{\mu\nu} = (G_{\mu\nu} - \eta_{\mu\nu})/2$ :

$$h_{\mu\nu}(t, \vec{x}) = \int_{\mathcal{C}} d^3x' \frac{T_{\mu\nu}(t - |\vec{x} - \vec{x}'|, \vec{x}')}{|\vec{x} - \vec{x}'|}, \quad (\text{B1})$$

where  $\mathcal{C}$  is the past-light cone and bodies have small polarized acceleration and angular momentum is small.

The integral equation may be solved by iteration in which a trial potential  $h_{\mu\nu} = 0$  is inserted into the integrand and the integral solved to arrive at a new potential  $h'_{\mu\nu}$  which is then reinserted to achieve a potential  $h''_{\mu\nu}$  and so on until the required accuracy is achieved. In order to derive the 1PN equations of motion general relativity requires two iterations but here we only require one, meaning that the 1PN equations for this theory are linear.

Let the baryon density be  $\rho_0$ , and the Newtonian potential is given by,

$$U = \int d^3x' \frac{\rho_0(t, \vec{x}')}{|\vec{x} - \vec{x}'|}. \quad (\text{B2})$$

Let  $U \sim \epsilon^2$  and  $v/c \sim \epsilon$  where  $v$  is the average velocity and  $c$  is the speed of light in vacuum. The first post-Newtonian corrections require that  $h_{00}$  be given up through order  $\epsilon^4$ ,  $h_{0j}$  through order  $\epsilon^3$  and  $h_{ij}$  through  $\epsilon^2$ , correcting the Newtonian order,

$$h_{00} = U, \quad h_{0j} = 0, \quad h_{ij} = 0. \quad (\text{B3})$$

The G-field derives from the equations of Sec. II C above and the stress-energy-momentum tensor for baryon dust. Each baryon particle or compact spherical mass (such

as a neutron star or black hole)  $p$  has a stress-energy-momentum tensor in its rest frame in a spherical coordinate system with the particle at the origin,

$$T_{00} = m_p \delta^3(\vec{x}), \quad T_{rr} = m_p \delta^3(\vec{x}). \quad (\text{B4})$$

The latter is related to the potential  $h_{rr}$  which has a transformation,

$$h_{ij} = h_{rr} \frac{x_i x_j}{|\vec{x}|^2}, \quad (\text{B5})$$

which is

$$h_{ij} = m_p \frac{(x_i - x'_i)(x_j - x'_j)}{|\vec{x} - \vec{x}'|^3} \quad (\text{B6})$$

for a particle at  $\vec{x}'$ . To obtain the potential for all the dust, sum all the potentials for all the baryons by superposition of the potentials,

$$\tilde{h}_{ij} = \int d^3x' \frac{\rho_0(t, \vec{x}') (x_i - x'_i)(x_j - x'_j)}{|\vec{x} - \vec{x}'|^3} + O(\epsilon^4). \quad (\text{B7})$$

By a non-Abelian gauge transformation [7],

$$h_{ij} = \tilde{G}_{ij} + \partial_i \theta_j, \quad (\text{B8})$$

where  $\theta_\mu = \partial_\mu \chi$  and

$$\chi = \int d^3x' \rho_0(t, \vec{x}') |\vec{x} - \vec{x}'|, \quad (\text{B9})$$

we have

$$h_{ij} = \delta_{ij} U + O(\epsilon^4). \quad (\text{B10})$$

The next two parts of the G-field,  $h_{0j}$  and  $h_{00}$ , require particle velocities and time dilation to be included. The current tensors,  $T_{\mu\nu}(p)$  of each baryon  $p$ , are in the rest frame, but, because the bodies are moving with respect to the observer at infinity and subject to the gravitational fields of one another, the tensors must be (1) boosted from comoving frame  $\bar{x}^\mu$  to the observer's frame,  $x^\mu$ , and (2) subject to a non-Abelian gauge transformation from the zero gravitational field (free fall) in the rest frame of the body to the gravitational field in the rest frame of the observer,

$$T_{\mu\nu}(p) = \frac{d\bar{x}^\mu}{dx^\alpha} \frac{d\bar{x}^\nu}{dx^\beta} T_{\alpha\beta}(p) + h_{\mu\nu} m_p \delta^3(\vec{x} - \vec{x}_p), \quad (\text{B11})$$

where  $\Lambda$  is a Lorentz transformation matrix for a boost density  $\vec{v}(t, \vec{x})$  ( $c = 1$ )[7] and  $h_{\mu\nu}$  is the G-field not including the contribution of the mass at that point. These two



transformations are equivalent to the two step, boost-and-coordinate-transform method of general relativity [7] which reflects the effects of both velocity and gravitational fields on measurements of stress-energy-momentum tensors.

For a baryon  $p$  with mass  $m_p$  at  $\vec{x}'$  with a rest frame potential  $\bar{h}_{\mu\nu}$  and velocity  $\vec{v}$  ( $c = 1$ ),

$$h_{0j} = \bar{h}_{00} \frac{\partial \bar{x}^0}{\partial x^0} \frac{\partial \bar{x}^0}{\partial x^j} + \bar{h}_{ij} \frac{\partial \bar{x}^i}{\partial x^0} \frac{\partial \bar{x}^j}{\partial x^j}, \quad (\text{B12})$$

where

$$\bar{h}_{ij} = \frac{m_p (x_i - x'_i)(x_j - x'_j)}{|\vec{x} - \vec{x}'|^3}. \quad (\text{B13})$$

To order this becomes,

$$\tilde{h}_{0j} = -\frac{m_p}{|\vec{x} - \vec{x}'|} \left( v_j + \frac{[(\vec{x} - \vec{x}') \cdot \vec{v}](x_j - x'_j)}{|\vec{x} - \vec{x}'|^2} \right) + O(\epsilon^5). \quad (\text{B14})$$

Under the same gauge transformation as above,

$$h_{0j} = \tilde{h}_{0j} - \partial_0 \theta_j, \quad (\text{B15})$$

and summing over all baryons, the potential is,

$$h_{0j} = -2V_j + O(\epsilon^5), \quad (\text{B16})$$

where

$$V_j = \int d^3x' \frac{\rho_0(t, \vec{x}') v_j(t, \vec{x}')}{|\vec{x} - \vec{x}'|}. \quad (\text{B17})$$

The time-time field is,

$$\tilde{h}_{00} = U + 2\Psi + \Phi + O(\epsilon^6), \quad (\text{B18})$$

where

$$\Psi(t, \vec{x}) = \int d^3x' \frac{\rho_0(t, \vec{x}') (v^2 + U)}{|\vec{x} - \vec{x}'|}, \quad (\text{B19})$$

and

$$\Phi(t, \vec{x}) = \int d^3x' \frac{\rho_0(t, \vec{x}')}{|\vec{x} - \vec{x}'|} \left\{ \frac{[(\vec{x} - \vec{x}') \cdot \vec{v}]^2}{|\vec{x} - \vec{x}'|^2} - v^2 \right\}. \quad (\text{B20})$$

The gauge transformation,  $h_{00} = \tilde{G}_{00} + \partial_0^2 \chi$  (which we have already applied to the other potentials and hence must carry over to this one), eliminates the last term, and gives

$$h_{00} = U + 2\Psi + O(\epsilon^6). \quad (\text{B21})$$

Missing is a term of order  $U^2$  that appears in metric PPN formalism which indicates the nonlinearity in the superposition of the potential. In GR, however, this term is an artifact of

the choice of isotropic coordinates and not a true nonlinearity such as would be associated with massive gravitons.

In general relativity, the change of coordinates,  $r = \bar{r}(1 + M/2\bar{r})^2$ , switches the Schwarzschild solution to isotropic coordinates [7],

$$ds^2 = - \left( \frac{1 - M/2\bar{r}}{1 + M/2\bar{r}} \right)^2 dt^2 + (1 + M/2\bar{r})^4 [d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2\theta d\phi^2)]. \quad (\text{B22})$$

Because the change in coordinates, however, depends on the potential itself, this introduces a nonlinearity that previously did not exist in the solution. Like the Schwarzschild solution in Schwarzschild coordinates, the Yang-Mills theory 1PN solution does not contain any nonlinearity because it is not in this nonlinear coordinate system. The exact, Yang-Mills spherically symmetric solution given in spherical coordinates,

$$h_{00} = h_{rr} = M/r, \quad (\text{B23})$$

under the same change of variables, becomes to 1PN order,

$$h_{00} = M/\bar{r} - 2(M/\bar{r})^2 + O(\epsilon^6), \quad (\text{B24})$$

$$h_{0i} = h_{i0} = O(\epsilon^5) \quad (\text{B25})$$

and

$$h_{ij} = \delta_{ij}M/\bar{r} + O(\epsilon^4). \quad (\text{B26})$$

While one could make a change of the flat spacetime coordinates to bring the YM potential in line with the GR metric, there is no purpose in doing so. Since both GR and the Yang-Mills theory are generally covariant, changing coordinates does not change predictions. It does, however, require that observations be input in the appropriate coordinate system since the Schwarzschild  $r$  is not the same as the 1PN  $\bar{r}$ . Indeed, because distances are defined by the metric,  $g_{\mu\nu}$ , for the YM theory  $r$  is the true distance between a point and the origin while, in General Relativity, the Schwarzschild  $r$  is simply a coordinate. Meanwhile, because of the potential dependent metric in “isotropic” coordinates,  $\bar{r}$  is just a coordinate in both theories and does not refer to distance directly.

The remaining equations (which can be derived from the field equations) are the equation of motion. The continuity equations (38) to 1PN order are the four equations,

$$\partial_\nu T^{\nu\alpha} = 0, \quad (\text{B27})$$

each of which is the same as the electromagnetic continuity equation. Linearized general relativity has the same problem [6] and care needs to be taken that the nonlinear, gauge covariant equations of motion (38) be used instead. For  $N$ -body problems, which are generally the ones that gravitation deals with, however, the geodesic equation is sufficient. This is because the Yang-Mills theory satisfies the Strong Equivalence Principle (SEP) and internal structure does not affect the motion of bodies. Therefore, any spherical, compact body moves as a test particle.